

UNIVERSITY OF ILLINOIS

DIGITAL COMPUTER

LIBRARY ROUTINE K 11 - 192

TITLE Quartimax Orthogonal Rotation of Factors (DOI Only)

DURATION .36 mp + ct = time in seconds

The first term is the time required for input and output where m is the number of factors, and p the number of variables in the matrix being rotated. The second term is the time required for computation. Only very rough estimates can be made of the number c of cycles required for convergence. As the number of factors increases, with a fixed number of variables, c increases rapidly. Centroid factors will usually require a larger c than principal axes factors. Approximate estimates of c may be taken from the following table:

NUMBER of FACTORS	4	6	8	12	16
c	4	7	12	21	36

The time t required for each cycle depends primarily on the number of factors and secondarily on the number of variables. The table below can be used for a rough estimation of t.

NUMBER of FACTORS	3	5	9	13	17	21
t (in seconds)	1	17	50	120	200	320

DESCRIPTION The aim of rotation may be taken as that of decreasing the complexity of the factorial description of the variables. The quartimax method of rotation provides for finding the orthogonal transformation which maximizes the variance of the squared factor loadings (thereby achieving a high degree of inequality among the factor loadings). This can be shown to be equivalent to maximizing the fourth powers of the loadings. In applying the method, two factors are operated upon at a time. The maximizing angle of transformation ϕ for two factors *i* and *j* is given by the equation:

time has elapsed, raise the black switch to OBEY. Wait for Illiac to stop on the order 24 090: Then start by raising white switch. Continuous punching will begin immediately and last less than 4 minutes.

Program ends on OF from 116.

CAPACITY (m) · (p) ≤ 745

COMMENTS At the end of each complete cycle of rotations the increase in the sum of fourth powers, $\sum_i \sum_j f_{ij}^4$, is inspected to see if another cycle of rotations should be made. If the increase has been less than $T \times 2^{-45} \times 10^4$ then no further iterations will be made, where T is an integer less than 1,000. The tolerance, T, is taken to be 10 unless specified. If the tolerance desired is not ten then insert the following immediately before the "22 278N" on the parameter tape:

00 29K
00 F 00 (T) F

If it is necessary to complete the rotation during a second time period on the computer, use the following procedure. The black switch is raised to the "obey" position about five minutes before the first time period is to end; the computer will complete a cycle of rotations and then stop. Start with the white switch as soon as the computer stops and the rotated factors will be printed. Proceed in the second session as in the first, but using the rotated factor tape as the matrix tape.

K11 contains an old obsolete version of T2 which does not agree with the serial copy of T2. This old version is correct, however.

JON: mge
RJT

RT: 10/13/60

DATE	November 17, 1955
PROGRAMMED BY	J. O. Neuhaus
CHECKED BY	R. J. Twery
APPROVED BY	J. P. Nash

This program was prepared for the library by the Psychology Department.

USE OF STORAGE - QUARTIMAX

<u>LOCATION</u>	<u>CONTENTS</u>
5	00 1F 00 1F
6	00 pF 00 pF : variables (on parameter tape)
7	40 $\binom{279}{+mp}$ F 00 F; $\cos \frac{\pi}{8}$
8	$\sin \frac{\pi}{8}$
9	$-\sin \frac{\pi}{8}$
10	previous $\sum_{i,j} f_{ij}^4$.
11	279 x 2 ⁻³⁹ during input; last $\sum_{i,j} f_{ij}^4$. during calculation
12	counter (accumulating variables)
13	i
14	j
15	binary switch (after forming sum)
16	$2\sum_k f_{ki}^2$
17	$2\sum_k f_{kj}^2$
18	$4\sum_k f_{ki}^3 f_{kj}$
19	$4\sum_k f_{ki} f_{kj}^3$
20	50 mp 00 F
21	temporary store for $\left(\sum_{k=1}^p f_{ki} f_{kj} (f_{ki}^2 - f_{kj}^2) \right) =$ Nu- merator
22	Denominator
23	binary switch
24	$1/2 \cos \frac{\pi}{8}$
25	$1/2 \sin \frac{\pi}{8}$
26	$\frac{\pi}{8}$

<u>LOCATION</u>	<u>CONTENTS</u>
27	m : factors (on parameter tape)
28	50 279F 75 279F
29	tolerance T
31	DNI Library Routine N - 3
52	SCP Library Routine P - 6
65	sin - cos Library Routine T - 1
95	arctan-Library Routine T - 2
124	body - Main Routine

LOCATION	ORDER	NOTES PAGE 1
Code X1		Decimal order input
00 11K		
00 F		
00 279F		
00 3K] Print parameter
00 F		
00 4F] tolerance
00 29K		
00 F		
00 10F		
00 5K		
00 1F		
00 1F		
00 7K		
40 SS		
00 F		
00 20K		
50 SS		
00 F		
00 24K		
00F00461939766200J		$1/2 \cos \frac{\pi}{8}$
00F00191341716150J		$1/2 \sin \frac{\pi}{8}$
00F00392699981699J		$\frac{\pi}{8}$
00 28K		
50 SS		
75 SS		
00 31K		
Library Routine N 3		Decimal number sequence input
00 52K		
Library Routine P 6		Single column print

LOCATION	ORDER	NOTES	PAGE 2
00 65K		Sine, cosine	
Library Routine T 1			
00 95K		Arctan	
Library Routine T 2			
	00 124K		
0	40 SS		
	50 L		
1	26 31F		
	15 L		
2	14 6F	Input factor matrix	
	46 L		
3	10 7F		
	36 5L		
4	26 L		
	41 10F		
5	50 SS		
	75 SS		
6	00 2F		
	40 F		
7	50 F		
	75 F		
8	00 2F		
	14 10F		
9	40 10F	Form sum of fourth powers ($\sum_{ij} f_{ij}^4$)	
	15 5L		
10	14 5F	test for convergence, store	
	40 5L		
11	10 20F	as N(10)	
	32 12L		
12	26 5L		
	92 515F		
13	15 11F		
	14 29F		
14	10 10F		
	32 16L		

LOCATION	ORDER	NOTES	PAGE 3
15	L5 10F 40 11F		
16	24 20L 92 59F		
17	92 59F 92 135F	Transfer to print routine if started with white switch	
18	26 143L 00 F		
19	00 F 00 F		
20	41 12F L5 5F	k → 0	
21	40 13F 40 14F	i, j → 1	
22	49 15F 26 23L		
23	50 SS 75 SS		
24	00 1F 40 F	increase 2 $\sum_k f_{ki}^2 = N(16)$	
25	L4 16F 40 16F		
26	50 SS 75 SS		
27	00 1F 40 1F	increase 2 $\sum_k f_{kj}^2 = N(17)$	
28	L4 17F 40 17F		
29	50 SS 75 SS		
30	40 2F 50 F		

LOCATION	ORDER	NOTES	PAGE 4
31	7J 2F L4 18F	increase $4 \sum_k f_{ki}^3 f_{kj} = N(18)$	Accumulate
32	40 18F 50 1F		Sums
33	7J 2F L4 19F	increase $4 \sum_k f_{ki}^3 f_{kj} = N(19)$	
34	40 19F L5 23L		
35	L4 5F 40 23L		
36	46 29L L5 26L		
37	L4 5F 40 26L	advance addresses	
38	42 29L L5 12F		
39	L4 5F 40 12F		
40	L0 6F 36 42L	test for completion of sums	
41	26 23L 00 F		
42	L5 18F L0 19F	store Numerator as N(19)	
43	40 21F L1 15F	test switch	
44	36 49L 49 23F	set switches	
45	41 15F L5 24F		
46	40 7F L5 25F		
47	40 8F L1 25F	store cos, sin, - sin of $\frac{\pi}{8}$ at 7, 8, 9	

LOCATION	ORDER	NOTES	PAGE 5
48	40 9F		
	26 87L		
49	L7 21F	test (Denon. - Numer.)	
	L2 22F		
50	36 51L		
	26 53L		
51	40 F	test (N = D)	
	L1 F		
52	32 82L		
	26 77L		
53	41 F		
	50 F		
54	L1 21F	$(-N) \div D$	
	66 22F		
55	S5 F		
	50 55L		
56	26 95F	find 1/4 arctan	
	10 2F		
57	40 F	test N	
	L5 21F		
58	36 72L	test D	
	L5 22F		
59	36 68L	test D - N	Find the angle
	L0 21F		of rotation θ
60	32 66L		
	L5 F		
61	L0 26F	find sin [θ]	
	50 61L		
62	26 65F		
	L5 2F		
63	40 7F		
	L5 1F		

LOCATION	ORDER	NOTES	PAGE 6
64	40 8F L1 1F	store cos, sin, - sin	
65	40 9F 41 23F		
66	26 87L L5 F		
67	L0 26F 26 61L		
68	L4 21F 36 71L		
69	L5 26F L4 26F		
70	L4 F 22 61L		Find the angle of rotation θ
71	23 69L 00 F		
72	L5 22F 32 75L		
73	L4 21F 32 74L		
74	22 60L L5 F		
75	22 61L L0 21F		
76	36 71L 22 74L		
77	41 F 50 F		
78	L1 22F 66 21F		
79	S5 F 50 79L		

LOCATION	ORDER	NOTES	PAGE 7
80	26 95F		
	10 2F		
81	40 F		
	L1 F		
82	26 57L		
	50 21F		
83	7J 22F		
	32 85L		
84	L1 26F		
	10 1F		
85	26 57L		
	L5 26F		
86	10 1F		
	26 57L		
87	41 2F		
	50 SS		
88	7J 7F		
	40 F		
89	50 SS		
	7J 8F		
90	L4 F		
	40 1F		
91	50 SS		
	7J 9F		
92	40 F		
	50 SS	find k^{th} component	
93	7J 7F	of the rotated vectors	
	L4 F		
94	40 3F		Rotate the i^{th}
	L4 3F		and j^{th} factors
95	40 SS		
	L5 1F		

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LOCATION	ORDER	NOTES	PAGE 8
96	L4 1F 40 SS		
97	F5 87L 42 87L		
98	42 96L 00 20F		
99	46 91L F5 92L		
100	42 92L 00 20F	reset addresses	
101	46 89L 46 95L		
102	L5 5F L4 2F		
103	40 2F L0 6F		
104	36 105L 22 87L		
105	41 16F 40 17F		
106	40 18F 40 19F	clear accumulators	
107	40 12F L1 23F	test switch	
108	36 118L L5 43L		
109	L4 5F 46 43L		
110	L5 23L L0 6F		
111	40 23L 46 29L		

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LOCATION	ORDER	NOTES	PAGE 9
112	42 87L	reset addresses in accumulation and rotation routines	
	42 96L		
113	46 91L		
	L5 26L		
114	L0 6F		
	40 26L		
115	42 29L		
	42 92L		
116	46 89L		
	46 95L		
117	26 23L	j → j + 1 test (i - m)	
	00 F		
118	L5 14F		
	L4 5F		
119	40 14F		
	L0 27F		
120	36 126L		
	L5 23L		
121	L0 6F		
	40 23L		
122	46 29L	accum. and rotation routines for j th factor	Reset addresses, increase counters, and test to find position in pro- gram
	42 87L		
123	42 96L		
	46 91L		
124	L5 43L		
	L0 5F		
125	46 43L		
	26 22L		
126	L5 13F		
	L4 5F		
127	40 13F	i → i + 1 test (i - m)	
	L0 27F		

LOCATION	ORDER	NOTES	PAGE 10
128	36 134L L5 13F		
129	L4 5F 40 14F		
130	L5 23L L4 6F	reset addresses for	
131	40 26L 42 29L	i th factor	
132	42 92L 46 89L		
133	46 95L 26 124L		
134	L5 28F 40 5L		
135	40 23L 46 29L		
136	42 87L 42 96L		
137	46 91L L4 6F	reset addresses to	
138	40 26L 42 29L	prepare for a new cycle	
139	42 92L 46 89L		
140	46 95L L5 43L		
141	L0 5F 46 43L		
142	22 4L 00 F		
143	41 7F 41 8F		

LOCATION	ORDER	NOTES	PAGE 11
144	L5 SS 50 144L		
145	26 52F L5 144L		
146	L4 5F 46 144L		
147	L5 8F L4 5F		Punch Rotated Factors
148	40 8F L0 6F		
149	36 150L 26 144L		
150	92 770F 92 129F		
151	L5 7F L4 5F		
152	40 7F L0 27F		
153	36 154L 22 143L		
154	0F F 41 11F		
155	50 6F 75 27F		
156	00 19F L4 20F	form mp	
157	46 20F 46 7F	and plant	Form dependent parameters
158	L5 23L L4 6F		
159	40 26L 42 29L	plant address of 1st	

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LOCATION	ORDER	NOTES	PAGE 12
<p>160</p> <p>161</p> <p>24 999N</p>	<p>42 92L</p> <p>46 89L</p> <p>46 95L</p> <p>24 L</p> <p>Sum check</p>	<p>loading in 2nd factor</p>	

K11

JON:
RJT: mge

11/17/55